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Superallowed $0^+ \longrightarrow 0^+$ β decay and CKM unitarity: A new overview including more exotic nuclei

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Abstract. The ft values for superallowed $0^+ \rightarrow 0^+$ nuclear β transitions are nearly independent of nuclear-structure ambiguities and depend uniquely on the vector part of the weak interaction. They thus provide access to clean tests of some of the fundamental precepts of weak-interaction theory and can even probe physics beyond the standard model. We have just completed a new survey and overview of world data on such transitions, including not just the nine cases considered in the past, but also eleven more from $T = 1$ parents: even-even $T_z = -1$ nuclei from ^{18}Ne to ^{42}Ti; and odd-odd nuclei from ^{62}Ga to ^{74}Rb. These new cases all involve more exotic nuclei and, to yield comparable precision, present real experimental challenges. Nevertheless, three of these new cases are already known well enough to contribute – together with the nine well known transitions – to the setting of limits on fundamental weak-interaction parameters. The remaining eight cases show promise for making important contributions in future.

PACS. 23.40.Bw Weak-interaction and lepton (including neutrino) aspects – 12.15.Hh Determination of Kobayashi-Maskawa matrix elements – 12.60.-i Models beyond the standard model

1 Introduction

The study of superallowed $0^+ \rightarrow 0^+$ β decay gives nuclear physicists access to some of the most fundamental properties of the weak interaction. It can be used to test the conservation of the vector current (CVC), set limits on any possible scalar currents, test the unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) matrix and probe the existence of right-hand currents. To be interesting, though, these tests demand high precision in the experimental determination of the transition ft values and, until recently, this has limited the study of superallowed decay to parent nuclei very near stability, where decay schemes are less complex and production rates are compatible with high counting statistics.

In the past, we have periodically surveyed relevant world data on such superallowed transitions and extracted from them the current best fundamental weak-interaction parameters. However, our most recent complete survey [1], which included eight well measured cases, was published in 1990 and since then, with the rise of radioactive-beam facilities, techniques have been developed that are now bringing even quite exotic nuclei into the realm of precision measurements. With this in mind, we have just completed a thorough new overview [2] in which we critically surveyed all relevant measurements, adjusted original data to take account of the most modern calibration standards, obtained statistically rigorous average results for each transition, and used updated and consistent calculations to extract weak-interaction parameters from those results. Although, there are still only eight transitions whose ft values are known to better than 0.1%, four more cases are now known with 0.1–0.4% precision. These four make real contributions to the usefulness of the overall results and several involve quite exotic nuclei. Finally, we identify a further eight transitions that are promising cases for future study.

2 Superallowed β decay

All superallowed transitions whose ft values have been measured with high precision are between $(J^\pi, T) = (0^+, 1)$ analog states. For such transitions, the measured ft-value can be related to the vector coupling constant *via* an expression that includes several small ($\sim 1\%$) correction terms. It is convenient to combine some of these terms with the ft-value and define a "corrected" $\mathcal{F}t$-value. Thus, we write [3]

$$\mathcal{F}t \equiv ft(1+\delta_R')(1+\delta_{NS}-\delta_C) = \frac{K}{2G_V^2(1+\Delta_R^V)}\,, \qquad (1)$$

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where $K/(\hbar c)^6 = 2\pi^3\hbar \ln 2/(m_e c^2)^5 = 8120.271(12) \times 10^{-10}\,\mathrm{GeV}^{-4}\mathrm{s}$, G_{V} is the vector coupling constant for semi-leptonic weak interactions, δ_C is the isospin-symmetry-breaking correction and $\Delta_{\mathrm{R}}^{\mathrm{V}}$ is the transition-independent part of the radiative correction. The terms δ'_R and δ_{NS} comprise the transition-dependent part of the radiative correction, the former being a function only of the electron's energy and the Z of the daughter nucleus, while the latter, like δ_C, depends in its evaluation on the details of nuclear structure. From this equation, it can be seen that each measured transition establishes an individual value for G_{V} and, if the CVC assertion is correct that G_{V} is not renormalized in the nuclear medium, all such values —and all the $\mathcal{F}t$-values themselves— should be identical within uncertainties, regardless of the specific nuclei involved.

The ft-value that characterizes any β-transition depends on three measured quantities: the total transition energy, Q_{EC}, the half-life, $t_{1/2}$, of the parent state and the branching ratio, R, for the particular transition of interest. The Q_{EC}-value is required to determine the statistical rate function, f, while the half-life and branching ratio combine to yield the partial half-life, t. In our treatment [2] of the data, we considered all measurements formally published before November 2004 and those we knew to be in an advanced state of preparation for publication by that date.

The final corrected $\mathcal{F}t$ values obtained for the best twelve cases from the survey [2] are plotted in the bottom panel of fig. 1. They cover a broad range of nuclear masses from $A = 10$ to $A = 74$. As anticipated by CVC (see eq. (1)) the $\mathcal{F}t$ values are statistically consistant with one another, yielding an average value $\overline{\mathcal{F}t} = 3072.7(8)\,\mathrm{s}$, with a corresponding chi-square per degree of freedom of $\chi^2/\nu = 0.42$. This expectation of CVC is thus verified at the level of 3×10^{-4}, which is the fractional uncertainty we obtain for $\overline{\mathcal{F}t}$. This is a 30% improvement over the best previous value [1] —also obtained from superallowed β decay— and can be attributed to improvements in the experimental data.

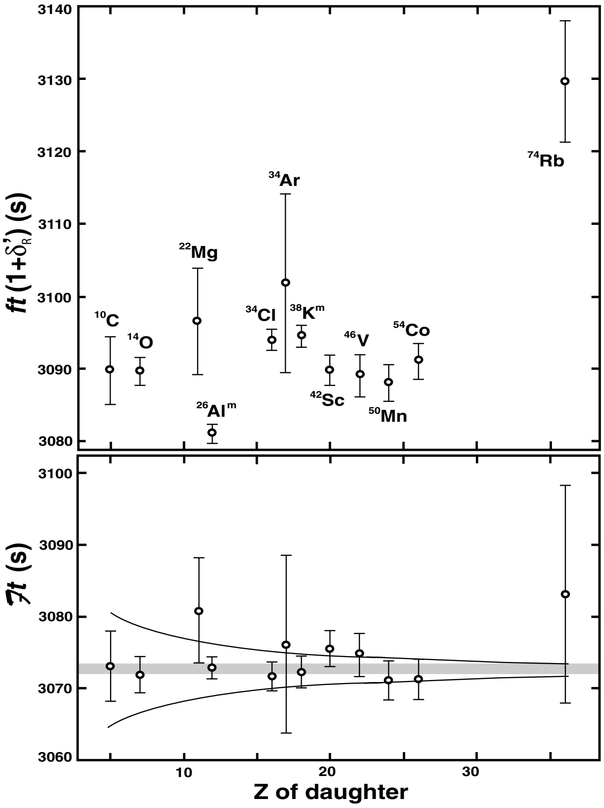

Fig. 1. In the top panel are plotted the experimental ft-values corrected only for δ'_R, those radiative effects that are independent of nuclear structure. In the bottom panel, the corresponding $\mathcal{F}t$ values are given; they differ from the top panel simply by inclusion of the nuclear-structure-dependent corrections, δ_{NS} and δ_C. (See eq. (1).) Note that the increased uncertainty for ^{74}Rb reflects the lack of experimental constraints on the nuclear shell model in this mass region. The horizontal grey band in the bottom panel indicates the average $\overline{\mathcal{F}t}$ value with its uncertainty. The curved lines represent the approximate loci the $\mathcal{F}t$ values would follow if an induced scalar current existed with $C_S/C_V = \pm 0.002$.

3 Weak-interaction tests

The data in the bottom panel of fig. 1 can also be analyzed to set a limit on the possible presence of scalar currents, which would affect the calculation of the statistical-rate function, f, *via* a term in the shape-correction function that is inversely proportional to the positron energy. Since the total superallowed-transition decay energy increases with Z, a scalar contribution would therefore have its greatest effect on the $\mathcal{F}t$ values at low Z, introducing curvature in that region. The curved lines in the figure are the loci of $\mathcal{F}t$ values that would be expected if $C_S/C_V = \pm 0.002$. Obviously, the $\mathcal{F}t$ values do not exhibit any such curvature and, from a least-square fit to the data, we obtain the limit $|C_S/C_V| \leq 0.0013$, as expressed in the conventional notation of Jackson, Treiman and Wyld [4]. The corresponding result for the Fierz interference constant is $b_F = +0.0001(26)$. These are, by far, the most stringent limits on $|C_S/C_V|$ or $|b_F|$ ever obtained from nuclear β decay.

With a mutually consistent set of $\mathcal{F}t$ values, we can now insert their average value into eq. (1) and determine the vector coupling constant G_{V} using the value $\Delta_{\mathrm{R}}^{\mathrm{V}} = 2.40(8)\%$ calculated for the transition-independent radiation correction by Marciano and Sirlin [5]. In doing so, we also make a small adjustment to the value of $\overline{\mathcal{F}t}$ to account for possible systematic uncertainties in δ_C by averaging our calculated δ_C values with those of Ormand and Brown [6] and increasing the assigned uncertainty (see ref. [2]). This leads to $\overline{\mathcal{F}t} = 3073.5(12)$, the result we carry forward. The derived value of G_{V} itself is of little interest but, when combined with G_{F}, the weak interaction constant for the purely leptonic muon decay, it yields a value for the element V_{ud} of the CKM matrix: $V_{ud} = G_{\mathrm{V}}/G_{\mathrm{F}}$. Taking the Particle Data Group (PDG) value [7] of $G_{\mathrm{F}}/(\hbar c)^3 = 1.16639(1) \times 10^{-5}\,\mathrm{GeV}^{-2}$, we obtain $|V_{ud}| = 0.9738(4)$. Compared to our previously recommended value [3], this result differs by two units in the last digit quoted and has a reduced uncertainty. Note that, by more than an order of magnitude, V_{ud} is the most precisely determined element of the CKM matrix.

The unitarity of the CKM matrix is a fundamental requirement of the standard model and the precise value for V_{ud} obtained from superallowed β decay is a key component of the most demanding test available of that unitarity. Combining our value for V_{ud} with the PDG's recommended values [7] of $V_{us} = 0.2200(26)$ and $V_{ub} = 0.00367(47)$, we obtain a unitarity sum for the top-row elements of the CKM matrix of

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9966 \pm 0.0014, \qquad (2)$$

which fails unitarity by 2.4 standard deviations. A recent measurement of the $K^+ \to \pi^0 e^+ \nu_e$ (K^+_{e3}) branching ratio from the Brookhaven E865 experiment [8] obtains $V_{us} = 0.2272 \pm 0.0030$. If this value alone were adopted for V_{us} rather than the PDG average of many experiments, the sum in eq. (2) would equal 0.9999(16) and unitarity would be fully satisfied.

Depending on what ultimately turns out to be the correct value for V_{us}, the result for the unitarity sum can be interpreted in terms of the possible presence of right-hand currents. We express the outcome in terms of the left-right and left-left coupling constants, a_{LR} and a_{LL}, defined by Herczeg [9]. If we accept the unitarity test result in eq. (2), then we find $Re a_{LR}/a_{LL} = -0.00176(74)$. Within the context of the manifest left-right symmetric model, this result corresponds to a mixing angle of $\zeta = 0.00176(74)$. If, instead, we adopt the E865 value for V_{us}, the result becomes $Re a_{LR}/a_{LL} = -0.00007(84)$.

4 Sharpening the tests in future

The accumulated world data on superallowed $0^+ \to 0^+$ β decay comprises the results of over one hundred measurements of comparable precision [2]. Virtually all the important experimental parameters used as input to the ft-value determinations have been measured in at least two, and often four or five, independent experiments. Obviously, just another measurement will not have much impact on the precision of the weak-interaction parameters quoted here. Nevertheless, it is still possible for well selected experiments with existing or currently foreseen techniques to make real improvements. For example, the bottom panel of fig. 1 clearly illustrates the sensitivity of the low-Z cases to the possible presence of scalar currents. Reduced uncertainties, particularly on the decays of ^{10}C and ^{14}O, could thus further reduce the scalar-current limit significantly.

It is the unitarity test, though, that is attracting the most interest at the moment, with various programs currently underway to improve this test. The appearance of one new measurement of the K_{e3} decay, which yielded quite a different result [8] for V_{us} from the previously accepted average, has stimulated other measurements of the same type. Within a few years, their results should settle the controversy over V_{us} and may, in themselves, bring eq. (2) into statistical agreement with unity. At the same time, experimentalists and theorists alike are seeking to improve the precision with which V_{ud} is known. Since the biggest contributor to the V_{ud} uncertainty —90% of it, in fact— comes from the calculation of $\Delta^{\mathrm{V}}_{\mathrm{R}}$, the top priority must be to improve that radiative-correction term, a difficult theoretical problem to which experiment cannot contribute. However, the next most important contributor is the structure-dependent correction terms, δ_{NS} and δ_C. These can actually be tested by experiment, and measurements of $0^+ \to 0^+$ β decays have been re-invigorated, now with a focus on the more exotic nuclei which particularly lend themselves to tests of the structure-dependent corrections.

The approach is best explained by a comparison of the top and bottom panels in fig. 1. The top panel shows a plot of $ft(1+\delta'_R)$, the experimental "raw" ft values corrected only for a structure-independent radiative effect. The corresponding $\mathcal{F}t$ values plotted in the bottom panel differ only by the application of the nuclear-structure-dependent corrections, $(\delta_C - \delta_{NS})$. Obviously, at the current level of precision the structure-dependent corrections act very well to remove the considerable "scatter" that is apparent in the experimental ft values and is effectively absent from the corrected $\mathcal{F}t$ values. It is important to note that the calculations of δ_{NS} and δ_C are based on well-established shell-model wave functions and were further tuned to measured binding energies, charge radii and coefficients of the isobaric multiplet mass equation [10]. Their origins are completely independent of the superallowed decay data. Thus, the consistency of the corrected $\mathcal{F}t$ values shown in fig. 1 is already a powerful validation of the calculated corrections used in their derivation.

The comparison in fig. 1 also suggests that the validation of $(\delta_C - \delta_{NS})$ can be further improved if future experiments focus on transitions with large calculated corrections. If the ft values measured for cases with large calculated corrections also turn into corrected $\mathcal{F}t$ values that are consistent with the others, then this must verify the calculation's reliability for the most precisely measured transitions, which have smaller corrections. The most potentially attractive cases for this purpose are in two series of 0^+ nuclei: the even-Z, $T_z = -1$ nuclei with $18 \le A \le 42$, (^{18}Ne, ^{22}Mg, ^{26}Si, ^{30}S, ^{34}Ar, ^{38}Ca, and ^{42}Ti) and the odd-Z, $T_z = 0$ nuclei with $A \ge 62$ (^{62}Ga, ^{66}As, ^{70}Br and ^{74}Rb). In fact, the most recent additions to the twelve transitions shown in fig. 1 —^{22}Mg, ^{34}Ar and ^{74}Rb— were chosen from these series and the latter two have large calculated corrections. Though their precision does not yet equal that of the others, their $\mathcal{F}t$ values do indicate that the corrections so far are living up to expectations. Reducing the uncertainties on these cases and adding new ones, such as ^{18}Ne, ^{30}S and ^{62}Ga, should be considered as high priority goals in improving the precision on V_{ud} and sharpening the unitarity test.

Certainly these new series of superallowed emitters present experimental challenges. In each case, both the parent and daughter nuclei are unstable, so each Q_{EC}

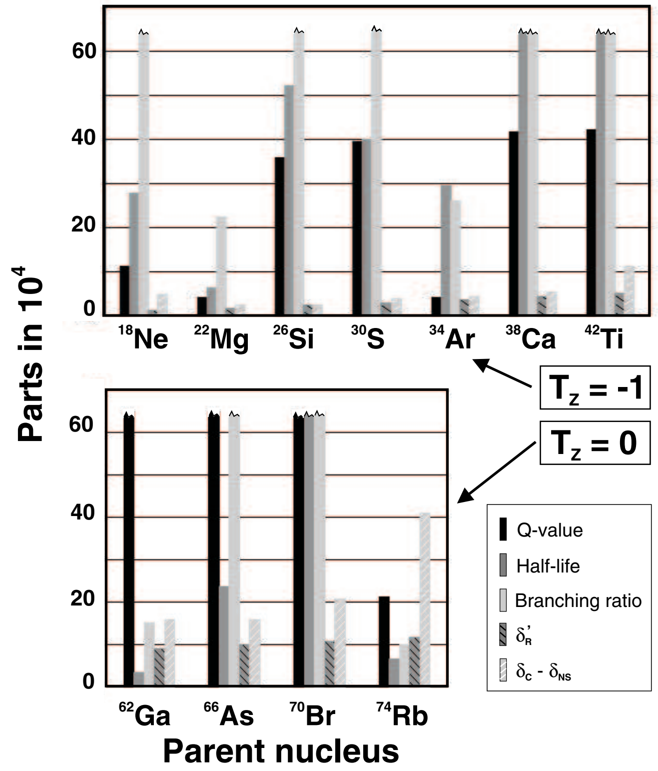

Fig. 2. Summary histogram of the fractional uncertainties attributable to each experimental and theoretical input factor that contributes to the final $\mathcal{F}t$ values for the "new" series of superallowed transitions currently under study. Where the uncertainty is shown as exceeding 60 parts in 10^4, no useful experimental measurement has been made.

value requires careful Penning-trap measurements of two masses. With half-lives, particularly of the heavier nuclei, pushing into the 100-ms range, the required $\sim$ 500-eV precision on these masses is at the very limit of what even Penning traps can achieve, at least in their present on-line configuration. Branching ratios too are stretching experimental limits. The superallowed transitions, dominant ($> 99.3\%$) branches for all but one of the "standard" superallowed cases, are seriously affected and sometimes dwarfed by competing Gamow-Teller branches in the new cases. For the $T_z = -1$ parents, which populate odd-odd daughters, there are only a few such competing branches in each case but they are strong —strong enough that the superallowed branch must be measured directly to 0.1% precision, a difficult task requiring unprecedented calibration standards in the measurement of β-delayed γ rays.

The new $T_z = 0$ emitters with $A \geq 62$ offer a different branching-ratio challenge. Their decays are of higher energy ($> 9\,\mathrm{MeV}$) and, although the daughters are even-even, their level density is high enough that numerous weak Gamow-Teller branches compete with the superallowed branch. Though the total Gamow-Teller strength can be significant, many of the individual branches are unobservably weak [11]. This "Pandemonium" effect (see ref. [12]) can be partially corrected for by careful measurement of weak β-delayed γ rays but ultimately one must rely on calculation to account for those γ rays that remain undetected.

Even with work on these new superallowed emitters only in its infancy, the ft values for ^{22}Mg, ^{34}Ar and ^{74}Rb have already been determined with remarkable precision (below ±0.4%). As these cases are improved and new ones added, the test of the nuclear-structure-dependent corrections will become more definitive. Figure 2 shows the present levels of uncertainties on the 11 transitions considered here as new superallowed transitions. Evidently much needs to be done but a very solid beginning has been made.

5 Conclusions

Our new survey of superallowed $0^+ \rightarrow 0^+$ decays has demonstrated the power of these data in probing some fundamental properties of the weak interaction. The CKM unitarity test, which deviates from unity by 2.4 standard deviations, is currently the most provocative outcome and future measurements of Q_{EC}-values, half-lives and branching ratios for the decays of some exotic nuclei have been proposed to help clarify this possible discrepancy.

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